

## Definition of GLCM Parameters

Sequence #	Parameter Name	Symbol	Definition	Property
1	Angular Second Moment (or energy or homogeneity)	ASM	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \{p(i, j)\}^2$	$G^{-2} \leq \text{ASM} \leq 1$
2	Contrast (or inertia)	CON	$\sum_{k=0}^{G-1} k^2 p_{x-y}(k)$	
3	Correlation	COR	$\frac{\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - \mu_x)(j - \mu_y) p(i, j)}{\sigma_x \sigma_y}$ $= \frac{\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (ij) p(i, j) - \mu_x \mu_y}{\sigma_x \sigma_y}$	$-1 \leq \text{COR} \leq 1$
4	Variance (or sum of squares)	VAR	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - \mu_x)^2 p(i, j)$ $= \sum_{i=0}^{G-1} (i - \mu_x)^2 p_x(i)$	
5	Inverse Difference Moment (or local homogeneity)	IDM	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{1}{1 + (i - j)^2} p(i, j)$	
6	Sum average (or mean value of 1 <sup>st</sup> and 2 <sup>nd</sup> gray-levels)	SAV	$\sum_{k=0}^{2G-2} k p_{x+y}(k) = \mu_x + \mu_y = 2\mu_x$	mean value of gray levels
7	Sum entropy	SEN	$-\sum_{k=0}^{2G-2} p_{x+y}(k) \cdot \log(p_{x+y}(k))$	$0 \leq \text{SEN} \leq \log(2G-1)$
8	Sum variance	SVA	$\sum_{k=0}^{2G-2} (k - \text{SAV})^2 p_{x+y}(k)$	
9	Entropy	ENT	$-\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} p(i, j) \cdot \log(p(i, j))$	$0 \leq \text{ENT} \leq \log(G^2)$
10	Difference entropy	DEN	$-\sum_{k=0}^{G-1} p_{x-y}(k) \cdot \log(p_{x-y}(k))$	$0 \leq \text{DEN} \leq \log G$
11	Difference variance	DVA	$\sum_{k=0}^{G-1} k^2 p_{x-y}(k) - \left(\sum_{k=0}^{G-1} k p_{x-y}(k)\right)^2$	
12	Dissimilarity	DIS	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1}  i - j  p(i, j)$	

13	Cluster shade	CLS	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i+j-\mu_x-\mu_y)^3 p(i,j)$ $= \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i+j-2\mu_x)^3 p(i,j)$	
14	Cluster prominence	CLP	$\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i+j-\mu_x-\mu_y)^4 p(i,j)$ $= \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i+j-2\mu_x)^4 p(i,j)$	
15	Maximum probability	MAP	$\max(p(i,j))$	

1. In the above table, we assume:

→ J(x, y) is a diffraction image before normalization

→ I(x, y) is the 8-bit diffraction image data after normalization by maximum and minimum pixel values of J

→ p(i, j) is an element of the GLCM of I(x, y)

→ Elements of the GLCM Matrix, p(i, j), are obtained as the averaged values of the corresponding elements from the four GLCM matrices calculated along 0°, 45°, 90° and 135° of pixel displacement vector **d** from the horizontal direction of the input image I.

2. Additional definitions of functions used in the above table with i or j = 0,1,2,...,G-1 and G as the number of gray levels (G=255 for 8-bit gray level image I):

$$p_x(i) = \sum_{j=0}^{G-1} p(i,j) = \text{sum of } i\text{th row} = \text{probability of } i \text{ as the first GL} \quad (1)$$

$$p_y(j) = \sum_{i=0}^{G-1} p(i,j) = \text{sum of } j\text{th column} = \text{probability of } j \text{ as the second GL} = p_x(j) \quad (2)$$

$$p_{x+y}(k) = \sum_{i=0}^{G-1} \sum_{\substack{j=0 \\ i+j=k}}^{G-1} p(i,j) = \text{sum of diagonal line with GL sum as } k$$

= probability of GL sum as k (3)

(k=0, 1, ..., 2G-2)

$$p_{x-y}(k) = \sum_{i=0}^{G-1} \sum_{\substack{j=0 \\ |i-j|=k}}^{G-1} p(i,j) = \text{sum of main diagonal line with GL difference as } k$$

= probability of GL difference as k (4)

(k=0, 1, ..., G-1)

Note that k (= 0, ..., G-1) measures the “distance” off the main diagonal of the GLCM p while  $p_{x-y}(k)$  provides a histogram of the absolute differences of gray levels from pixel pairs of input image I.

$$\mu_x = \sum_{i=0}^{G-1} i p_x(i) = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} i p(i,j) = \text{mean value of 1st GL} \quad (5)$$

$$\sigma_x = \sqrt{\sum_{i=0}^{G-1} (i - \mu_x)^2 p_x(i)} = \text{standard deviation of 1}^{\text{st}} \text{ GL in GL pairs} \quad (6)$$

$$\mu_y = \sum_{j=0}^{G-1} j p_y(j) = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} j p(i, j) = \text{mean value of 2nd GL} = \mu_x \quad (7)$$

$$\sigma_y = \sqrt{\sum_{j=0}^{G-1} (j - \mu_y)^2 p_y(j)} = \text{standard deviation of 2nd GL} = \sigma_x \quad (8)$$

3. SAV can be shown to be the mean value of 1<sup>st</sup> and 2<sup>nd</sup> gray-levels and twice of MEA defined in early version of this note

$$SAV = \sum_{k=0}^{2G-2} k p_{x+y}(k) = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i + j) p(i, j) = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} i p(i, j) + \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} j p(i, j) = \sum_{i=0}^{G-1} i p_x(i) + \sum_{j=0}^{G-1} j p_y(j) = \mu_x + \mu_y = 2\mu_x$$

$$MEA = \sum_{i=0}^{G-1} i \sum_{j=0}^{G-1} p(i, j) = \sum_{i=0}^{G-1} i p_x(i) = \mu_x = \frac{SAV}{2}$$