Definition of GLCM Parameters

Sequence #	Parameter Name	Symbol	Definition	Property
1	Angular Second Moment （or energy or homogeneity）	ASM	$\sum_{i=0}^{G-1}\sum_{j=0}^{G-1}\{p(i,j)\}^2$	$G^{-2} \leq$ ASM ≤ 1
2	Contrast (or inertia)	CON	$\sum_{k=0}^{G-1}k^2 p_{x-y}(k)$	
3	Correlation	COR	$\frac{\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(i-\mu_x)(j-\mu_y)p(i,j)}{\sigma_x\sigma_y} = \frac{\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(ij)p(i,j)-\mu_x\mu_y}{\sigma_x\sigma_y}$	-1 ≤ COR ≤ 1
4	Variance (or sum of squares)	VAR	$\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(i-\mu_x)^2 p(i,j) = \sum_{i=0}^{G-1}(i-\mu_x)^2 p_x(i)$	
5	Inverse Difference Moment (or local homogeneity)	IDM	$\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}\frac{1}{1+(i-j)^2}p(i,j)$	
6	Sum average (or mean value of 1st and 2nd gray-levels)	SAV	$\sum_{k=0}^{2G-2}kp_{x+y}(k)=\mu_x+\mu_y=2\mu_x$	mean value of gray levels
7	Sum entropy	SEN	$-\sum_{k=0}^{2G-2}p_{x+y}(k)\cdot\log(p_{x+y}(k))$	0 ≤ SEN ≤ log(2G-1)
8	Sum variance	SVA	$\sum_{k=0}^{2G-2}(k-SAV)^2 p_{x+y}(k)$	
9	Entropy	ENT	$-\sum_{i=0}^{G-1}\sum_{j=0}^{G-1}p(i,j)\cdot\log(p(i,j))$	0 ≤ ENT ≤ log(G^2)
10	Difference entropy	DEN	$-\sum_{k=0}^{G-1}p_{x-y}(k)\cdot\log(p_{x-y}(k))$	0 ≤ DEN ≤ logG
11	Difference variance	DVA	$\sum_{k=0}^{G-1}k^2 p_{x-y}(k)-(\sum_{k=0}^{G-1}kp_{x-y}(k))^2$	
12	Dissimilarity	DIS	$\sum_{i=0}^{G-1}\sum_{j=0}^{G-1}\lvert i-j\rvert p(i,j)$	

13	Cluster shade	CLS	$\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(i+j-\mu_x-\mu_y)^3 p(i,j)$ $=\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(i+j-2\mu_x)^3 p(i,j)$	
14	Cluster prominence	CLP	$\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(i+j-\mu_x-\mu_y)^4 p(i,j)$ $=\sum_{i=0}^{G-1}\sum_{j=o}^{G-1}(i+j-2\mu_x)^4 p(i,j)$	
15	Maximum probability	MAP	$\max(p(i,j))$	

1. In the above table, we assume:
   → J(x, y) is a diffraction image before normalization
   → I(x, y) is the 8-bit diffraction image data after normalization by maximum and minimum pixel values of J
   → p(i, j) is an element of the GLCM of I(x, y)
   → Elements of the GLCM Matrix, p(i, j), are obtained as the averaged values of the corresponding elements from the four GLCM matrices calculated along 0°, 45 °, 90 ° and 135° of pixel displacement vector **d** from the horizontal direction of the input image I.

2. Additional definitions of functions used in the above table with i or j = 0,1,2,…,G-1 and G as the number of gray levels (G=255 for 8-bit gray level image I):

$$p_x(i)=\sum_{j=0}^{G-1}p(i,j)=\text{sum of ith row}=\text{probability of i as the first GL} \quad (1)$$

$$p_y(j)=\sum_{i=0}^{G-1}p(i,j)=\text{sum of jth column}=\text{probability of j as the second GL}=p_x(j) \quad (2)$$

$$p_{x+y}(k)=\sum_{\substack{i=0\\i+j=k}}^{G-1}\sum_{j=0}^{G-1}p(i,j)=\text{sum of diagonal line with GL sum as k}$$
$$=\text{probability of GL sum as k} \quad (3)$$
$$\text{(k=0, 1, ..., 2G-2)}$$

$$p_{x-y}(k)=\sum_{\substack{i=0\\|i-j|=k}}^{G-1}\sum_{j=0}^{G-1}p(i,j)=\text{sum of main diagonal line with GL difference as k}$$
$$=\text{probability of GL difference as k} \quad (4)$$
$$\text{(k=0, 1, ..., G-1)}$$

Note that k (= 0, …, G-1) measures the "distance" off the main diagonal of the GLCM p while $p_{x-y}(k)$ provides a histogram of the absolute differences of gray levels from pixel pairs of input image I.

$$\mu_x=\sum_{i=0}^{G-1}ip_x(i)=\sum_{i=0}^{G-1}\sum_{j=0}^{G-1}ip(i,j)=\text{mean value of 1st GL} \quad (5)$$

$$\sigma_x=\sqrt{\sum_{i=0}^{G-1}(i-\mu_x)^2 p_x(i)}=\text{standard deviation of }1^{st}\text{ GL in GL pairs} \quad (6)$$

$$\mu_y = \sum_{j=0}^{G-1} j p_y(j) = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1} j p(i,j) = \text{mean value of 2nd GL} = \mu_x \tag{7}$$

$$\sigma_y = \sqrt{\sum_{j=0}^{G-1} (j-\mu_y)^2 p_y(j)} = \text{standard deviation of 2nd GL} = \sigma_x \tag{8}$$

3. SAV can be shown to be the mean value of 1st and 2nd gray-levels and twice of MEA defined in early version of this note

$$SAV = \sum_{k=0}^{2G-2} k p_{x+y}(k) = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1} (i+j) p(i,j) = \sum_{i=0}^{G-1}\sum_{j=0}^{G-1} i p(i,j) + \sum_{i=0}^{G-1}\sum_{j=0}^{G-1} j p(i,j) = \sum_{i=0}^{G-1} i p_x(i) + \sum_{j=0}^{G-1} j p_y(j) = \mu_x + \mu_y = 2\mu_x$$

$$MEA = \sum_{i=0}^{G-1} i \sum_{j=0}^{G-1} p(i,j) = \sum_{i=0}^{G-1} i p_x(i) = \mu_x = \frac{SAV}{2}$$